Introduction

It has been our privilege to present Calculus Connections at two previous ICTCM conferences, first in Orlando (1994 [1]) when the project was in its early stages, then at Houston (1995 [2]) when Volume 1 was complete. Volume 2 is now complete, and in this paper we review progress and give some examples of content from Volume 2.

Calculus Connections is a joint project between ourselves and two US organisations, the publisher J. Wiley & Sons, and software house Intellipro Inc. Calculus Connections will be a 3 volume CD-ROM multimedia package for PC/Windows, with 8 modules plus a Laboratory Manual for each volume.

It is important to re-iterate what we are not trying to do. This is not a package aimed at replacing teachers or lecturer; it is enrichment material. Also, we do not intend that Calculus Connections should replace the use of a standard textbook. In fact it can be used with any existing course whether it is "traditional" or "reformed", provided that some time is available for learning activities other than traditional "chalk and talk".

Our consistent aim throughout the project has been to use the computer to do the things it does well (interaction, graphics, etc.), and avoid using it for things for which it is not best adapted (for example, delivering large amounts of text, especially mathematical text). In Calculus Connections we use multimedia to capture attention and set up real world situations in which mathematics can be applied. Then we use simulations and interactive graphics to explain how the mathematics can be applied to these situations. The overall intention is to motivate the student to see the point of studying maths, and to provide strong visual associations (mental coat-pegs) on which to hang recollections of the relevant theory; this is particularly aimed at those students who find mathematics unrelated to the real world at best and irrelevant at worst. To reinforce these learning opportunities, we provide a printed Lab Book containing more mathematical details, with further explanations and exercises.

Calculus Connections structure

The package uses the modular approach. When complete there will be 24 modules, published as 3 volumes of 8 modules each on a CD-ROM, plus one Lab Book per volume. Volumes 1 and 2 have already been published. The modules cover topics from a typical US calculus course over three semesters, ranging from basic ideas about functions and slopes of tangents, through differentiation, integration, to three
dimensional calculus and spherical and polar co-ordinates.

Modules in Volume 1, already published:

| 1. Lines, Functions and Equations | 5. Applied Maximums & Minimums |
| 2. Limits | 6. Areas as limits |
| 3. Rates of change & differentiation | 7. Fundamental Theorem of Calculus |
| 4. Inverse & transcendental functions | 8. Mean Value Theorem |

Modules in Volume 2, already published:


Modules in Volume 3, planned:

| 17. Functions of Two or More Variables | 21. Double and Triple Integrals in Cylindrical and Spherical Coordinates |
| 18. Vector valued functions | 22. Centroids etc., and Moments of Inertia |

The material has been structured with short laboratory sessions in mind. For the student we have provided a simple structure of small blocks of material so that it is easy to navigate and intuitively simple to work out where to go and what to do next; however, the choice is left to the student. For the instructor we have made it easy to make assignments which encourage students of all abilities to explore calculus and its connections to the real world. The aim of the Lab book is to provide background mathematical theory, set exercises and more extensive problems.

**A brief look at Calculus Connections, Volume 2**

Each module of Calculus Connections is built around two "real life" applications which are first presented to the student using multimedia technology. These are intended to be followed by key mathematical concepts, and a number of interactive exercises. A high level of interaction encourages the student to investigate the nature of mathematical models, and explore their relationship with the real world. The material is usually studied in that order, but the student can access any part of the package at will. Other resources include a 3-D graph plotter, glossary, mathematical biographies, on-line help about using the package, and links to computer algebra packages. There is an accompanying Laboratory manual which has more extensive textual material and extended exercises. This combination allows course instructors a great deal of freedom to exploit Calculus Connections to best advantage for their own students.

At the start of the package, a menu is displayed listing the modules available. Clicking on the module of choice then brings up a the module menu, as shown for example in Figure 1. The student is thus presented at a very early stage with a clear screen which encapsulates the design philosophy of the package. At the end of each block of material (i.e. application, concept or exercise), the software returns to the module menu.
The application blocks all follow a pattern. First there is a video clip with a voice-over which sets out a (reasonably) everyday situation which is of some practical importance. For example, the population growth application in Module 13 "Differential Equations" raises the question as to how it is possible to model population growth: how do we construct differential equations to do this, and what do they predict? This introduces the need for suitable techniques to study differential equations. The scene is now set to move from video to animation, which is used to demonstrate how numerical data from real life can be represented mathematically and graphically, and so help the learner to appreciate the connection between these ideas. Whereas a video is non-interactive (except that it can be started, stopped or replayed), animations permit some interaction: for example, the user can alter the parameters of the equation and see that the resulting solution can fit actual US Population Census data (see Figure 2). The overall aim of an application block is to provide a reference point for a mathematical technique or concept, and to give a flavour of what it is and how it works.

Concept blocks are intended to explain mathematical ideas in more detail. They never do this merely by putting text on the screen. Indeed, one of the purposes of the Lab Book is to provide a vehicle for presenting detailed mathematical results in the way
that most users find clearest: on paper. The intention is to develop a theme in a way which requires the learner to interact with the software. However, it would be unhelpful to put no mathematics at all on screen, as it provides the context for the interaction. This is not the place to go into all the types of interaction that are possible (although as noted in [3] there must be a rich field of interesting evaluation research questions to investigate here). The dominant use is exploring the effect of parameter changes, plotting graphs, and entering new functions. Figure 3 shows a typical example, where the user can explore the properties of a first order differential equation using direction fields. Note that as well as there being pre-set examples, there is always an opportunity for users to explore problems of their own choosing. In this case an arbitrary mathematical expression for $\frac{dy}{dx}$ can be entered.

The exercises are similar in screen style to the concept blocks, but they are designed to be done once the concepts have been understood. They use self-assessment in the sense that the courseware indicates whether each answer is correct or not. If an answer is wrong, the user can ask to see the correct answer or ask for a hint.

![Figure 3: exploring differential equations using direction fields](image)

**Mathematical Modeling: Module 16**

The objective of much of the mathematics taught is to enable us to model real situations by mathematical equations; in Module 13 we modeled population growth by a differential equation. In Module 16 we look at the modeling process itself. One problem which is considered is a model of the vertical oscillation of a car.

![Figure 4: Schematic drawing of a simple model for a car suspension](image)

The imbalance between the forces in the suspension, $T$, and the mass of the car being supported, $mg$, results in a net vertical acceleration. Newton's Laws of motion give
where \( y(t) \) is the displacement of the car. In the module the user is encouraged to explore the relationship between \( T \) and \( y \) by a simple animation, and should be able to deduce that \( T = k(L - y) \) where \( k \) is a spring stiffness and \( L \) is the natural length of the suspension. The solution of the resulting equation is \( y(t) = A \sin(\omega t + \phi) + B \). The power of the PC is now available to explore the effect of the constants, \( A \), \( \omega \), \( \phi \), and \( B \) and how they relate to any initial state of the car. However, we can take this process further by asking whether the assumption that \( T = k(L - y) \) is essential; could \( T = ky \) or \( T = k(L - y)^{-1} \) lead to similar solutions?

**Evaluation, Future Plans and Conclusions**

Although large numbers of copies of Volume 1 have been distributed, it has proved difficult to obtain significant feedback from users. A major priority for the coming year is to make contact with users so that a survey can be conducted, using questionnaires. Any user who reads this and would like to take part is welcome to contact the authors.

In the meanwhile we have continued to rely on detailed reviewing from a team of US College professors, organised by our publishers. We should like to again record our thanks for their extremely valuable help.

One of us (DAQ) has also been able to make significant use of Volume 1 in his teaching as part of a foundation year course for students who intend to read Mathematics or Science subjects which would normally require a standard "A" level entry requirement. The course covers the basic ideas of the Differential and Integral Calculus and is taught in 3 strands. The first is a single lecture each week which lays down a skeleton of the material being studied; this is expanded upon by the students who use other computer based modules developed by the UKMCC [4]. Calculus Connections is used to both motivate and also to assess the students. At relevant points in the semester the relevant modules are made available for self study and the students complete the exercises in the Lab Book. The marks achieved then count towards the
overall continuous assessment mark, which itself is a significant contribution towards the course mark of each student. It is hoped to analyse the performance and attitudes of this year's students when the course is complete.

The first two volumes of Calculus Connections have taken the project from elementary ideas at the foundation of Calculus, up to the point where significant practical use can be made of mathematics in modeling some simple dynamical systems, where quantities of interest are functions of just one variable. In the third volume, we will be tackling more advanced concepts which require the calculus of two or more variables. We think that interactive graphics will offer significant help to those many students who find this difficult. We hope to be able to report progress in a year's time.

References


